Fuzzy Multiple Objective Models For Facility Location Problems

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Abstract: There are a variety of efficient approaches to solve crisp multiple objective decision making problems. However in the real life the input data may not be precisely determined because of the incomplete information. This paper deals with a multi objective facility location problem using the algorithm developed by Drezner and Wesolowski.

Keywords: Fuzzy decision making, multi objective decision, fuzzy goal programming, facility location problem.

1. INTRODUCTION

In a standard multiple goal programming, goals and constraints are defined precisely. Fuzzy goal programming has the advantage of allowing for the vague aspirations of decision makers, which are quantified by some natural language rules [1-18].

To our knowledge, first R. Narasimhan [15] introduced fuzzy set theory into objective programming. Since then many achievements have been added to the literature. In the following, an approach for solving fuzzy multiple goal problems will be presented, and its application to a facility location problem will be discussed.

2. MULTIPLE FUZZY GOAL PROGRAMMING

In a multiple goal programming problem, the optimal realization of multiple objectives is desired under a set of constraints imposed by a real life environment. If the goals and constraints are all expressed with equalities, we have a completely symmetric formulation

Find $\mathbf{x}$ such that $\mathbf{Ax} = \mathbf{b},\ \mathbf{x} \geq \mathbf{0}$. (1)

Where $\mathbf{x}$ is the vector of variables, $\mathbf{b}$ is the vector of the goals and available resources, and $\mathbf{A}$ is the matrix of the coefficients. In the cases when the decision maker is not precise in goals and restrictions, the linguistic statements such as "around **b**" will be used. In this case the above crisp goal programming problem becomes

$$\text{Find } \mathbf{x} \text{ such that}$$
$$\mathbf{Ax} = \tilde{\mathbf{b}},\ \mathbf{x} \geq \mathbf{0}. \qquad (2)$$

Where the fuzzy components b_i of the fuzzy vector **b** can be represented by, for example, triangular fuzzy numbers:

$$\mu_i(z) = \begin{cases} (z-(b_i-d_{i1}))/d_{i1},\ b_i-d_{i1} \leq z \leq b_i, \\ ((b_i+d_{i2})-z)/d_{i2},\ b_i \leq z \leq b_i+d_{i2}, \\ 0, \quad \text{elsewhere.} \end{cases} \qquad (3)$$

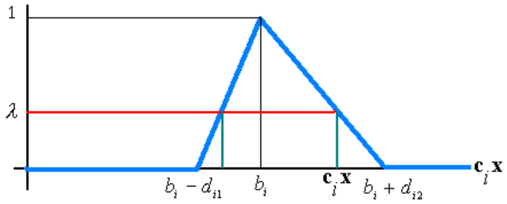

Figure 1. Fuzzy components b_i of the fuzzy vector **b.**

To have a membership number at least λ, $\mathbf{c}_i\mathbf{x}$ must remain in the interval

$$b_i - d_{i1} + \lambda d_{i1} \leq \mathbf{c}_i\mathbf{x} \leq b_i + d_{i2} + \lambda d_{i2} \qquad (4)$$

that is

$$(\mathbf{c}_i\mathbf{x} - (b_i - d_{i1}))/d_{i1} \geq \lambda,$$
$$((b_i + d_{i2}) - \mathbf{c}_i\mathbf{x})/d_{i2} \geq \lambda. \qquad (5)$$

Hence the above fuzzy goal programming problem is the maximum satisfaction problem of the fuzzy equations, and this goal can be achieved by the solution of the below crisp linear programming problem Lai, and Wang [14].

$$\text{Max } \lambda \text{ such that for all } i,$$
$$(\mathbf{c}_i\mathbf{x} - (b_i - d_{i1}))/d_{i1} \geq \lambda,$$
$$((b_i + d_{i2}) - \mathbf{c}_i\mathbf{x})/d_{i2} \geq \lambda. \qquad (6)$$

3. A FACILITY LOCATION PROBLEM

Bhattacharya, J.R. Rao, and R.N. Twari [2] have used fuzzy goal programming to locate a single facility on a plane bounded by a convex polygon under three objectives:

i. Maximize the minimum distances,
ii. Minimize the maximum distances from the facilities to the demand points,
iii. Minimize the sum of all transport costs.

Let $P_i = (a_i, b_i),\ i = 1,2,\ldots,m$ be the locations of demand points, $S = (x, y)$ is the location of the new facility, and X is the set of feasible points for new facility. Then,

$$\text{Max } g_1(x,y) = \min_i (|x-a_i| + |y-b_i|)$$
$$\text{Min } g_2(x,y) = \max_i (|x-a_i| + |y-b_i|)$$
$$\text{Min } g_3(x,y) = \Sigma_i w_i (|x-a_i| + |y-b_i|) \quad (7)$$

Such that
$c_{j1}x + c_{j2}y \leq c_{j3},\ j = 1,2,\ldots,n,\quad (x,y) \in X.$

Where w_i's denote the cost per unit distance between the new facility S and demand points $P_i = (a_i, b_i)$. To describe the distances the taxicab geometry or city block distance is used since the problem is considered in an urban area. Euclidean distance could also be used.

The same problem can also be formulated as follows:

Find $S = (x, y)$ such that

$$
\begin{aligned}
& g_1 \ge g_1^0 \\
& g_2 \le g_2^0 \\
& g_3 \le g_3^0 \qquad\qquad\qquad\qquad (8) \\
& |x-a_i|+|y-b_i| \ge g_1,\ \forall i \\
& |x-a_i|+|y-b_i| \le g_2,\ \forall i \\
& c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n, \\
& (x,y)\in X.
\end{aligned}
$$

where g_1^0, g_2^0, g_3^0 are the three goals prescribed by the decision maker. One may use positive ideal solution (g_1^*, g_2^*, g_3^*) to represent the goals and tolerances of fuzzy goals may be the differences of the positive and negative ideal solutions (g_1^-, g_2^-, g_3^-).

Positive and negative ideal solutions are the solutions of the following problems:

g_1^* : Max g_1 such that

$$
\begin{aligned}
& |x-a_i|+|y-b_i| \ge g_1,\ \forall i \\
& c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n,\ (x,y)\in X.
\end{aligned}
$$

g_1^- : Min g_1 such that

$$
\begin{aligned}
& |x-a_i|+|y-b_i| \ge g_1,\ \forall i \\
& c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n,\ (x,y)\in X.
\end{aligned}
$$

g_2^* : Min g_2 such that

$$|x-a_i|+|y-b_i| \le g_2,\ \forall i$$

$$c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n,\ (x,y)\in X.$$

g_2^- : Max g_2 such that

$$|x-a_i|+|y-b_i| \le g_2,\ \forall i$$

$$c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n,\ (x,y)\in X.$$

g_3^* : Min g_3 such that

$$|x-a_i|+|y-b_i| \le g_3,\ \forall i$$

$$c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n,\ (x,y)\in X.$$

g_3^- : Max g_3 such that

$$
\begin{aligned}
& |x-a_i|+|y-b_i| \le g_3,\ \forall i \qquad\qquad (9) \\
& c_{j1}x+c_{j2}y \le c_{j3},\ j=1,2,\dots,n,\ (x,y)\in X.
\end{aligned}
$$

Then using the values of (g_1^*, g_2^*, g_3^*) and (g_1^-, g_2^-, g_3^-), the fuzzy limitations for the goals $(\tilde{g}_1, \tilde{g}_2, \tilde{g}_3)$ are obtained as follows.

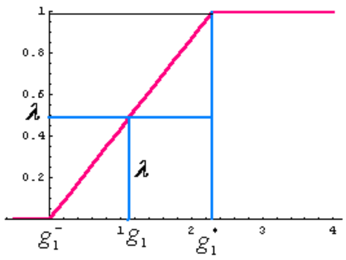

Figure 2. Fuzzy goal $\tilde{g}_1$.

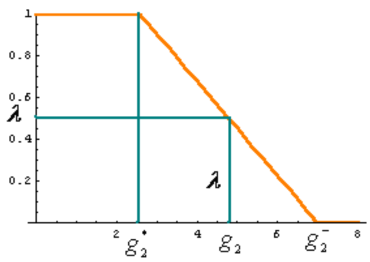

Figure 3. Fuzzy goal $\tilde{g}_2$.

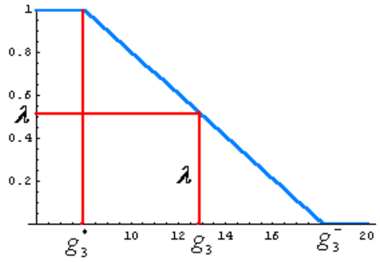

Figure 4. Fuzzy goal $\tilde{g}_3$.

With these fuzzy goals, problem (8) can be expressed as

Find $S = (x, y)$ Such that

$$g_1 \geq \tilde{g}_1,\; g_2 \leq \tilde{g}_2,\; g_3 \leq \tilde{g}_3 \tag{10}$$

$$|x - a_i| - |y - b_i| \geq g_1,\; \forall i,$$

$$|x - a_i| - |y - b_i| \leq g_2,\; \forall i$$

$$c_{j1}x + c_{j2}y \leq c_{j3},\; j = 1,2,\ldots,n,\; (x, y) \in X.$$

To transform the problem (10) into a crisp problem with only one objective, we get the $\lambda -$cuts:

Maximize λ such that

$$g_1 - g_1^- \geq \lambda(g_1^* - g_1^-),$$

$$-g_2 + g_2^- \geq \lambda(-g_2^* + g_2^-),$$

$$-g_3 + g_3^- \geq \lambda(-g_3^* + g_3^-) \tag{11}$$

$$|x - a_i| + |y - b_i| \geq g_1,\; \forall i,$$

$$|x - a_i| + |y - b_i| \leq g_2,\; \forall i$$

$$c_{j1}x + c_{j2}y \leq c_{j3},\; j = 1,2,\ldots,n,\; (x, y) \in X.$$

3. AN APLICATION

Let $\{P_i\} = \{(0, 2), (2, 1), (3, 4)(1, 3.5)(2.5, 2)(2, 4)(1, 0)(.5, .5)\}$ be the locations of the eight demand points, $S = (x, y)$ is the location of the new facility. Let unit costs per unit distance between the new facility S and the demand point $P_i(a_i, b_i)$ be $\{w_i\} = \{0.7, 2.1, 1.5, 1., 1., 1., 1., 1.\}$. In this case the values of (g_1^*, g_2^*, g_3^*) are found to be $(1.5, 3.0, 19.0)$ and the values of (g_1^-, g_2^-, g_3^-) are found to be $(0.0, 7.0, 35.2)$.

Hence in the feasible region $X = \{(x, y) | 4x + 57 \leq 20, 8x + 3y \leq 24, x, y \geq 0\}$, the problem (16) becomes:

Maximize λ such that

$$g_1 \geq 1.5\lambda,\; g_2 \leq 7 - 4.\lambda,\; g_3 \leq 35.2 - 16.2\lambda$$

$$x + y - g_1 \geq c3,\; x - y + g_1 \leq c2,$$

$$x + y + g_1 \leq c1$$

$$x + y + g_2 \geq c1,\; x - y + g_2 \geq c2,$$

$$x + y - g_2 \geq c3,\; x - y - g_2 \geq c4 \tag{12}$$

$$4x + 5y \leq 20,\; 8x + 3y \leq 24.\; x, y \geq 0.$$

Where

$$c1 = \max_i (a_i + b_i),\; c2 = \max_i (a_i - b_i),$$

$$c3 = \min_i (a_i + b_i),\; c4 = \min_i (a_i - b_i).$$

The solution of the above problem is found to be $x = 1.696,\; y = 2.642,\; \lambda = 0.831$. For this optimum supply point $S = (1.31, 2.23)$, one has

$$g_1(x, y) = 1.419,\; g_2(x, y) = 3.458,\; g_3(x, y) = 20.838. \tag{13}$$

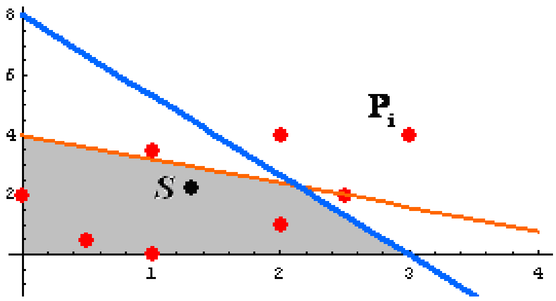

Figure 5. The three demand points $\{P_i\} = \left\{ \begin{matrix} (0, 2), (2, 1), (3, 4)(1, 3.5) \\ (2.5, 2)(2, 4)(1, 0)(.5, .5) \end{matrix} \right\}$, and the new optimum supply point $S = (1.31, 2.23)$.

4. DISCUSSION

In this article, to deal with and optimization problem with three fuzzy objectives, a method due to Narasimhan, R. is used. The maximum and minimum solutions of each sub problems are found, and then using this information, goals are transformed into fuzzy inequalities. Then a crisp symmetric optimization problem is obtained by $\lambda -$cuts. The number of the demand points can be increased to represent a real world problem easily.

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