Southeast Europe Journal of Soft Computing ISSN 2233 – 1859
Available online: http://scjournal.ius.edu.ba

Vol 12, No. 2, September 2023, pp. 21-24

Few techniques of stability analysis for infectious disease employ the compartmental model

[1*]Nagadevi Bala Nagaram, [1]Kala Raja Mohan [1] R.Narmada Devi and [1]Suresh Rasappan

[1]Department of Mathematics,
Veltech Rangarajan Dr. Sagunthala R&D Institute of Science and Technology,
400 feet Outer Ring Road,
Tamil Nadu, 600062
[1]Mathematics Section, Information Technology Department,
University of Technology and Applied Sciences- Ibri,
Postal Code 516,
Sultanate of Oman
**Corresponding Author*:nagaramnagadevibala@gamil.com

Article Info

Article history:

Article received on 09 10 2023

Received in revised form 29 10 2023

Keywords:

Routh-Hurwitz; health; compartmental models; mathematical models

ABSTRACT: The cycle of life includes everything from joy to sorrow to good health to sickness. Most people have had a viral infection at some point in their lives, whether it was a small infections or the flu. It is amazing that a microscopic particle that cannot even be seen under a microscope and cannot even replicate on its own can enter any living creature and use the resources of that life form to create thousands of copies of the virus, some of which can even be fatal to the living species. Understanding the origin, means of prevention, means of control, and attempts at preventative measures are essential in the fight against these illnesses. It is immoral to experiment on infectious diseases, unlike other types of research. On the other hand, mathematical models can reasonably explain how the disease is spreading. This article focuses on few compartmental models and a technique to analyze the infectious disease. The control analysis technique which employ to comprehend how diseases move among the populations and where the controls are required. In this article Routh-Hurwitz criterion is employed to analyze the system of equations.

1. INTRODUCTION

The body is depicted as a single compartment in the most basic scenario, such as a one-compartment model. In systems including chemical reactions, biological processes, and ecological interactions, compartment models are frequently used to simulate the transit of material. They are made up of a number of compartments connected by a variety of material fluxes.

Early 20th-century works by Ross[1] in 1916, Ross and Hudson in 1917,[2][3], Kermack and McKendrick in 1927[4], and Kendall in 1956 are significant examples of the origins of these models[5]. The Reed-Frost model was a crucial and frequently disregarded forebear of contemporary methods for epidemiological modelling[6].

The simplest method to explain how drugs are distributed throughout the body and eliminated is with the one-compartment open model. This model presupposes that the drug is able to penetrate and leave the human organism (i.e., that the model is "open"), that the entire body behaves as a single, uniform entity. The simplest method to explain how drugs are distributed throughout the body and eliminated is with the one-compartment open model. According to this concept, the body functions as a single, uniform compartment and the drug can enter or exit the body. This model is open compartment.

In two compartmental model,the whole population can divide into two parts and analysis can done. It is known as a very basic SI model. Another the most basic compartmental model is the SIR model, from which many other models are derived. Compartmental modelling is a very adaptable modelling tool. They are frequently used in the by employing mathematical modelling of infectious diseases. Labelled compartments are used to define the population, such as S, I, or R (Susceptible, Infectious, or Recovered). The transmission between compartments is possible. The labels' arrangement often indicates the flow patterns between the compartments; for instance, SEIS stands for susceptible, exposed, infected, then susceptible once more. In such way we can express different compartmental mathematical model as per required to analysis the infectious disease.

In section 2, the different types of compartmental model are presented with diagram. In section 3 the Routh Hurwitz technique is discussed whether the system is stable or unstable. In section 4, the numerical simulation for basic compartmental models are presented follows the conclusion

2. PROBLEM STATEMENT

Consider a system of equations with characteristic equation. We have no assurance that the system will be stable even if all the coefficients have the same sign and there are no missing terms. To do this, we apply the Routh Hurwitz Criterion to assess the system's stability. The system is said to be unstable if the predetermined conditions are not met. The authors A. Hurwitz and E.J. Routh provide this criterion. The advantage of this method is to obtain without solving the equation, we can determine whether the system is stable. We can check the stability of the system without solving the equation.

3. BACKGROUND

1.1. One compartmental model

Leon Shargel, Susanna Wu-Pong, Andrew B.C. Yu[7] presented the one compartmental model which is the easiest approach to explain how drugs are distributed and eliminated from the body is with the one-compartment open model. The body is treated as a single, uniform compartment in this model, which operates under the assumption that the drug can enter or exit the body. They explained that the oral route, in which dosage forms (drug products) like tablets, capsules, or oral solutions are typically utilized, is both the most popular and the most preferred method of medication delivery. A pharmacokinetic model must take into account both the route of administration and the pharmacokinetic behaviour of the medication in the body in order to characterize and predict drug disposition kinetically. A pharmacokinetic model can predict drug disposition after which dosage regimens for specific patients or groups of patients can be calculated. In such a way we can define a process through a one compartmental model. The following Figure 1 represent the one compartmental model. The rate of injected drug in body is take in account as αD_ib, the distribution of drug in the body is considered as S_ib and β refers here the constant elimination rate.

Figure 1:OCM Model

1.2. Two Compartmental Model:SI or SIS Model

Of all the disease models, the SI model is the most easy to understand the infectious disease. People by birth can take into account as susceptible without any immunity. Without treatment, those who become infected remain contagious for the rest of their lives and continue to interact with those who are vulnerable. The HIV infectious disease is fall into this SI or SIS model. The following Figure2 expressed the SI or SIS model. S is susceptible and I refers to Infectious state.

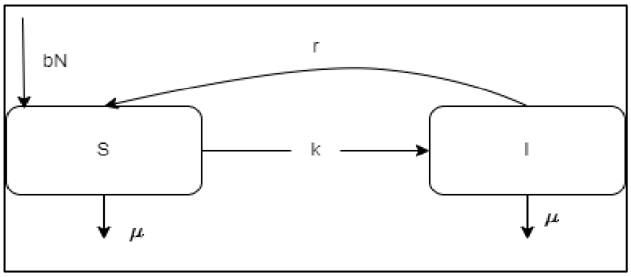

Figure 2: SI or SIS Model

The total population have taken into two states as susceptible and infectious state . Hence N=S+I.

The tansmission rate from susceptible to Infectious state is k and Infectious state to Susceptible state is r. μ is death rate at each state. bN is the population of susceptible from the population.

The system of equations is defined as follow:

$$\frac{dS}{dt} = bN + (k - \mu)S + rI \qquad (1)$$

$$\frac{dI}{dt} = kS + (\mu - r)I \qquad (2)$$

The deterministic model is described by the following figure:

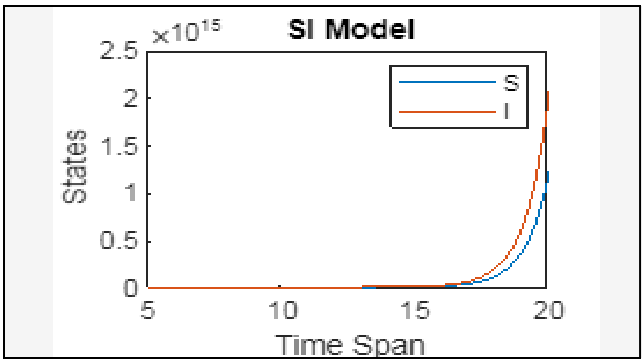

Figure 3:

To control the infectious state need to stabilize the transmission rate from susceptible to infectious state. So we can adopt the Lyapunov function to stabilize the state through Local and Global stability analysis.

According to the Routh Hurwitz criterion, a system can only be stable if and only if all of the roots in the first column have the same sign. If there are sign changes in the first column, the number of sign changes is equal to the number of roots in the characteristic equation in the right half of the s-plane, or the number of roots with positive real parts.

1.3. Multi Compartmental Model

A multi-compartment model is a type of mathematical model used for describing the transmitted among the compartments of a system. There are other variations on the SIR model, such as those that take into account births and deaths, recovery without immunity (SIS model), immunity lasting just a short time (SIRS model), and a latent period of the illness during which the person is not contagious. Multiple risk groups can also be modelled using compartmental models. SEIR model is important compartmental model. The SEIR model, which we have used to compute the number of infected, recovered, and dead individuals on the basis of the number of contacts, probability of disease transmission, incubation and infectious periods, and disease fatality rate, is one of the most frequently used mathematical algorithms to describe the diffusion of an epidemic disease. The following figure shows the SEIR model.

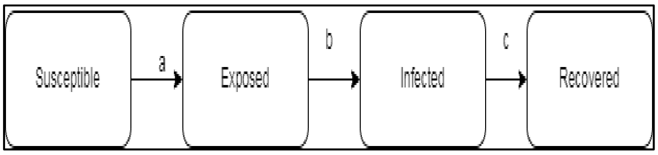

Figure 4:

The stability analysis is taken to control the required states if required to stabilize the system or to know the nature of the system.

1.4. Routh – Hurwitz Criterion:

Consider a system of equations with characteristic equation. We have no assurance that the system will be stable even if all the coefficients have the same sign and there are no missing terms. To do this, we apply the Routh Hurwitz Criterion to assess the system's stability. The system is said to be unstable if the predetermined conditions are not met. The authors A. Hurwitz and E.J. Routh provide this criterion[8]. The advantage of this method is to obtain without solving the equation, we can determine whether the system is stable. We can find the stability of the system without solving the equation.

First examine the stable, unstable, and marginally stable systems before talking about the Routh-Hurwitz Criterion. A system is deemed to be stable if all of the characteristic equation's roots are located on the left side of the "S" plane. System that is marginally stable is one in which all of its roots are located on the hypothetical axis of the "S" plane. A system is considered to be unstable if all of its roots are located on the right side of the "S" plane. To fulfill certain requirements in order to stabilize any system first consider the characteristic equation with all coefficients have same sign and no terms are missing in the equation. Consider the two compartmental model system of equation:

$$\frac{dS}{dt} = bN + (k - \mu)S + rI \qquad (1)$$

$$\frac{dI}{dt} = kS + (\mu - r)I \qquad (2)$$

The linearized matrix differential form for the system of equation is given as

A= $\begin{pmatrix} k-\mu & r \\ k & \mu - r \end{pmatrix}$ whose characteristic equation is

$\lambda^2 + (r-k)\lambda + \{k\mu - 2kr - \mu^2 + \mu r\} = 0$, where $(r-k) > 0$ and $\{k\mu - 2kr - \mu^2 + \mu r\} > 0$.

First arrange all the coefficients of the above equation in two rows:

Row 1 a_0=1 a_2= $k\mu - 2kr - \mu^2 + \mu r$

Row 2 a_1=r-k a_3=0

Then from these two row we can find the 3rd row as follows:

Row 1 a0=1 a2= $k\mu - 2kr - \mu^2 + \mu r$

Row 2 a1=r-k a3=0

Row 3 b1=-{ $k\mu - 2kr - \mu^2 + \mu r$ }

Where b1=$-\frac{1}{r-k}\begin{vmatrix} 1 & k\mu - 2kr - \mu^2 + \mu r \\ r-k & 0 \end{vmatrix}$

It reflects that the system is unstable. So to stabilize we can have techniques like feedback, backstepping analysis etc.

4. CONCLUSION

The control analysis technique which employ to comprehend how diseases move among the populations and where the controls are required. In this article Routh-Hurwitz criterion is employed and obtained the system is unstable for two compartmental model.

REFERENCES

[1] Harko T, Lobo FS, Mak MK (2014). "Exact analytical solutions of the Susceptible-Infected-Recovered (SIR) epidemic model and of the SIR model with equal death and birth rates". Applied Mathematics and Computation. 236: 184–194.

[2] R.M. Anderson, H.C. Jackson, R.M. May, A.M. Smith: Population dynamics of fox rabies in Europe. Nature, 289, 765–771 (1981).

[3]. R.M. Anderson, R.M. May: Infectious Diseases of Humans. Oxford Science Publications, Oxford (1991).

[4]. F. Brauer, C. Castillo-Chavez: Mathematical Models in Population Biology and Epidemiology. Springer, New York (2001).

[5]. S. Busenberg, K.L. Cooke: Vertically Transmitted Diseases: Models and Dynamics. Biomathematics, Vol. 23. Springer, Berlin Heidelberg New York (1993).

[6]. C. Castillo-Chavez with S. Blower, P. van den Driessche, D. Kirschner, A.A. Yakubu (eds.): Mathematical Approaches for Emerging and Reemerging Infectious Diseases: An Introduction. Springer, Berlin Heidelberg New York (2001).

[7]. P. van den Driessche, J. Watmough: Reproduction numbers and sub-threshold endemic equilibria for compartmental models of disease transmission. Math. Biosci., 180, 29–48 (2002).

[8]. Bodson, Marc, "Explaining the Routh–Hurwitz Criterion: A Tutorial Presentation [Focus on Education]". IEEE Control Systems Magazine. 40 (1): 45–51. doi:10.1109/MCS.2019.2949974. ISSN 1941-000X (2020).