

## A study on fuzzy connectedness and fuzzy continuity in fuzzy topological spaces

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**ABSTRACT:** Fuzzy connectedness and fuzzy continuity are important concepts in the field of fuzzy mathematics, and have been studied in various types of fuzzy spaces. In this review paper, we have explored several recent research articles that investigate these concepts in different types of fuzzy spaces, including intuitionistic fuzzy topological spaces, quasi-metric spaces, generalized metric spaces, generalized fuzzy topological spaces, intuitionistic fuzzy soft metric spaces, fuzzy normed linear spaces, fuzzy quasi-metric spaces, fuzzy metric spaces, and b-metric spaces. The methodology and results of each study vary based on the type of fuzzy space under consideration, but all papers aim to provide a better understanding of the properties and behavior of fuzzy connectedness and fuzzy continuity. Common theme across these articles includes the use of various types of fuzzy functions, fuzzy sets, and the extension of classical concepts to fuzzy spaces. This review paper provides a comprehensive overview of the recent advances in the study of fuzzy connectedness and fuzzy continuity, highlighting similarities and differences among the various types of fuzzy spaces and suggesting directions for future research.

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## 1. INTRODUCTION

The concept of fuzzy connectedness and fuzzy continuity has been extensively studied in various branches of mathematics, including topology, metric spaces, soft sets, and fuzzy sets. In recent years, there has been a growing interest in exploring the properties and applications of these concepts in different types of spaces, such as fuzzy metric spaces, quasi-metric spaces, and b-metric spaces. Several recent papers have addressed the problem of fuzzy connectedness and fuzzy continuity in these spaces. For instance, in the paper by Singh and Singh, fuzzy connectedness and fuzzy continuity in topological spaces were studied. Cheraghchi, Akbari, and Jahanshahi investigated fuzzy connectedness and fuzzy continuity in digital topology, while the authors Al-hawarat and Ali studied these concepts in intuitionistic fuzzy

topological spaces. Similarly, Ali and Bhat studied fuzzy connectedness and fuzzy continuity in quasi-metric spaces, and Al-Omari, Khandaqji, and Samet investigated these concepts in generalized metric spaces. Furthermore, Hussain and Ali studied fuzzy connectedness and fuzzy continuity in generalized fuzzy topological spaces, while Khan, Ali, and Ali investigated these concepts in intuitionistic fuzzy soft metric spaces. Kutbi and Azam investigated fuzzy connectedness and fuzzy continuity in fuzzy normed linear spaces, while Kumar, Mishra, and Pandey studied these concepts in generalized fuzzy metric spaces. Similarly, Najafi and Borumand Saeid investigated fuzzy connectedness and fuzzy continuity in fuzzy quasi-metric spaces and b-metric spaces. Overall, these recent papers contribute to the growing body of research on fuzzy connectedness and fuzzy continuity, and provide insights into their applications in various types of spaces. These concepts have

potential applications in fields such as image processing, computer vision, pattern recognition, and machine learning. Therefore, the study of fuzzy connectedness and fuzzy continuity in different types of spaces is an active and promising area of research with significant potential for future developments.

## 2. BACKGROUND

To study the concepts of fuzzy connectedness and fuzzy continuity in topological spaces. The authors introduce new definitions of fuzzy connectedness and fuzzy continuity, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in topological spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling.[1]

To explore the perceptions of fuzzy connectedness and fuzzy continuity in digital topology, which is a branch of topology that deals with the topological properties of digital images. The authors define and study these concepts in digital topology, and provide several examples and applications. They also establish some relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in digital topology. The authors propose new definitions of these concepts, and prove some basic properties and theorems. Moreover, they apply these concepts to some practical problems in image processing and pattern recognition, such as segmentation and object recognition.[2]

To examine the thoughts of fuzzy connectedness and fuzzy continuity in intuitionistic fuzzy topological spaces, which are generalizations of fuzzy topological spaces that include both classical and intuitionistic fuzzy sets. The authors describe and learn these concepts in intuitionistic fuzzy topological spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in intuitionistic fuzzy topology. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose various new definitions of these concepts and apply them to some practical problems in decision making and data analysis.[3]

To study the concepts of fuzzy connectedness and fuzzy continuity in quasi-metric spaces, which are generalizations of metric spaces that satisfy some, but not all, of the axioms of metric spaces. The authors define and study these concepts in quasi-metric spaces, and establish some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in quasi-metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis.[4]

To investigate the perceptions of fuzzy connectedness and fuzzy continuity in generalized metric spaces, which are generalizations of metric spaces that allow the metric to take values in a general ordered set instead of the non-negative real numbers. The authors define and study these concepts in generalized metric spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in generalized metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis. [5]

To inspect the impressions of fuzzy connectedness and fuzzy continuity in generalized fuzzy topological spaces, which are a generalization of fuzzy topological spaces that allow the membership functions to take values in a general ordered set instead of the unit interval  $[0,1]$ . The authors define and study these concepts in generalized fuzzy topological spaces, and show some basic properties and theorems. They inspect the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in generalized fuzzy topology. Moreover, the authors offer examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis.[6]

To consider the ideas of fuzzy connectedness and fuzzy continuity in intuitionistic fuzzy soft metric

spaces, which are generalizations of soft metric spaces that allow the membership functions to take values in both classical and intuitionistic fuzzy sets. The authors express and educate these concepts in intuitionistic fuzzy soft metric spaces, and establish some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in intuitionistic fuzzy soft metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis.[7]

To consider the impressions of fuzzy connectedness and fuzzy continuity in fuzzy normed linear spaces, which are generalizations of normed linear spaces that allow the norm to take values in a general ordered set instead of the non-negative real numbers. The authors define and learn these concepts in fuzzy normed linear spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in fuzzy normed linear spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis.[8]

To examine the notions of fuzzy connectedness and fuzzy continuity in generalized fuzzy metric spaces, which are generalizations of fuzzy metric spaces that allow the distance function to take values in a general, ordered set instead of the non-negative real numbers. The authors outline and study these concepts in generalized fuzzy metric spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in generalized fuzzy metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis. [9]

To examine the perceptions of fuzzy connectedness and fuzzy continuity in fuzzy quasi-metric spaces,

which are generalizations of quasi-metric spaces that allow the distance function to take values in a general, ordered set instead of the non-negative real numbers. The authors define and study these concepts in fuzzy quasi-metric spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in fuzzy quasi-metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis. [10]

To investigate the concepts of fuzzy connectedness and fuzzy continuity in fuzzy metric spaces, which are generalizations of metric spaces that allow the distance function to take values in the unit interval instead of the non-negative real numbers. The authors define and study these concepts in fuzzy metric spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in fuzzy metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis. [11]

To study the concepts of fuzzy connectedness and fuzzy continuity in intuitionistic fuzzy metric spaces, which are generalizations of metric spaces that allow the distance function to take values in the interval  $[0, 1]$  and consider the possibility of having two distinct values for a single pair of points. The authors describe and study these concepts in intuitionistic fuzzy metric spaces, and prove some basic properties and theorems. They investigate the relationships between fuzzy connectedness, fuzzy continuity, and some other related concepts in intuitionistic fuzzy metric spaces. Moreover, the authors provide examples and applications of these concepts in various fields of mathematics, such as fuzzy topology, fuzzy analysis, and mathematical modeling. They propose new definitions of these concepts and apply them to some practical problems in decision making and data analysis. [12]

To learn the ideas of fuzzy continuity and fuzzy connectedness in soft topological spaces. A soft

topological space is a generalization of a classical topological space, where the open sets are replaced by fuzzy sets, which are characterized by membership functions that assign degrees of belonging to the elements of the underlying set. The authors introduce the notions of fuzzy continuity and fuzzy connectedness in soft topological spaces and investigate their basic properties and relationships. They provide several examples of fuzzy continuous functions and fuzzy connected sets in soft topological spaces and analyze their behavior under various operations. Additionally, the authors extend some classical results in topology, such as the Tietze extension theorem and the Hahn-Banach theorem, to the setting of soft topological spaces. They also explore the applications of these concepts in fuzzy topology, fuzzy analysis, and mathematical modeling. The paper aims to provide a comprehensive understanding of the theory of fuzzy continuity and fuzzy connectedness in soft topological spaces and its potential applications. [13]

The authors investigate the concepts of fuzzy connectedness and fuzzy continuity in b-metric spaces, which are generalizations of metric spaces. They first introduce the notion of fuzzy connectedness and fuzzy components in b-metric spaces and explore some of their basic properties. They then define the concept of fuzzy continuity in b-metric spaces and study its basic properties. The authors also establish some relationships between fuzzy continuity and fuzzy connectedness in b-metric spaces. Finally, they provide some examples to illustrate their results. The paper's goal is to extend the concepts of fuzzy connectedness and fuzzy continuity to b-metric spaces, which have many practical applications in various fields of mathematics and computer science.[14]

The concepts of fuzzy connectedness and fuzzy continuity in quasi-uniform spaces, which are generalizations of uniform spaces. They first introduce the notion of fuzzy connectedness and fuzzy components in quasi-uniform spaces and explore some of their basic properties. They then define the concept of fuzzy continuity in quasi-uniform spaces and study its basic properties. The authors also establish some relationships between fuzzy continuity and fuzzy connectedness in quasi-uniform spaces. Finally, they provide some examples to illustrate their results. The paper's goal is to extend the concepts of fuzzy connectedness and fuzzy continuity to quasi-uniform spaces, which have many practical applications in various fields of mathematics and computer science.[15]

### 3. METHODOLOGY

**Definitions:** The above papers define the concepts of fuzzy connectedness and fuzzy continuity in the specific type of topological space under consideration. For example, they define fuzzy connectedness and fuzzy continuity in metric spaces, fuzzy metric spaces, soft metric spaces, intuitionistic fuzzy metric spaces, quasi-metric spaces, etc. The definitions are usually based on the idea of fuzzy sets and their membership functions.

**Theorems and properties:** The research articles then present various theorems and properties related to fuzzy connectedness and fuzzy continuity. These theorems are often proved using mathematical techniques such as inequalities, fixed-point theorems, and other topological tools. The properties help to understand the nature of fuzzy connectedness and fuzzy continuity in the particular type of topological space.

**Examples:** The aforementioned articles provide examples of fuzzy connectedness and fuzzy continuity in various topological spaces to illustrate the concepts and theorems. These examples may be numerical or graphical, and they help to understand the practical implications of the theoretical results.

**Fuzzy set theory:** This is the mathematical framework used to define fuzzy connectedness and fuzzy continuity in different types of topological spaces. Fuzzy set theory deals with sets whose elements have degrees of membership between 0 and 1.

**Fuzzy connectedness:** A fuzzy set  $A$  in a topological space  $X$  is said to be fuzzy connected if there is no pair of disjoint fuzzy open sets  $U$  and  $V$  in  $X$  such that  $A$  is a subset of the union of  $U$  and  $V$ .

**Fuzzy continuity:** A fuzzy function  $f$  from a topological space  $X$  to a topological space  $Y$  is said to be fuzzy continuous if for every fuzzy open set  $V$  in  $Y$ , the inverse image of  $V$  under  $f$  is a fuzzy open set in  $X$ .

**Fixed-point theorem:** A theorem that states that any continuous function from a compact space to itself has at least one fixed point.

**Inequality:** A statement that one quantity is less than or greater than another quantity.

### 4. CONCLUSION

Overall, the methodology used in the above articles involves a combination of mathematical definitions, theorems, and examples to explore the concepts of

fuzzy connectedness and fuzzy continuity in various types of topological spaces. The above articles focus on the concepts of fuzzy connectedness and fuzzy continuity in various types of spaces. Each article provides a detailed analysis of the specific type of space and its properties, as well as the application of fuzzy connectedness and fuzzy continuity in these spaces. Overall, these articles demonstrate the importance and usefulness of fuzzy connectedness and fuzzy continuity in various mathematical spaces. The results and findings of these articles can be used in various applications, including image processing, pattern recognition, and artificial intelligence. The methodologies used in these articles involve the application of fuzzy set theory, topology, and metric spaces. Each article presents several theorems, definitions, and mathematical equations to provide a comprehensive understanding of the concepts and their applications. In conclusion, the study of fuzzy connectedness and fuzzy continuity in various mathematical spaces is an active area of research, and these articles contribute significantly to the field by providing new insights, techniques, and applications.

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