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2-Domination Polynomial of Tensor Product of Paths

[1]Sanal Kumar, [2]Wasim Raja

[1]Sanal.Kumar@ibrict.edu.om, University of Technology and Applied Sciences(UTAS) Ibri, Oman

[2]Wasim.Raja@ibrict.edu.om, University of Technology and Applied Sciences(UTAS) Ibri, Oman

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ABSTRACT: Consider a simple finite graph G. The 2-domination polynomial for any simple non isolated graph G in [7] and is defined by $D_2(G,x) = \sum_{i=\gamma_2(G)}^{|V(G)|} d_2(G,i)\, x^i$, where $d_2(G,i)$ represents cardinality of 2-dominating sets of size i of graph G, and γ_2(G) is the 2-domination number of G. We have calculated the 2-domination number of the tensor product of P_2 and P_n. We have derived the 2-distance domination polynomials of tensor product of P_2 and P_n.

1. Introduction

Consider $G = (P, Q)$ be a simple graph having n vertices.Where P represent the vertices and Q represent edges of graph G. A subset $S \subseteq P$ is a 2-dominating set of the graph G, if every vertex $v \in P - S$ is adjoining to at least 2 vertices of S. The minimum cardinality of the 2-dominating sets in G is the $2 -$ domination number of a graph G, denoted by $\gamma_2(G)$. The smallest integer greater than or equal to n is represented by the notation $\lceil n \rceil$ in this paper.

2. 2-Distance Domination Polynomial

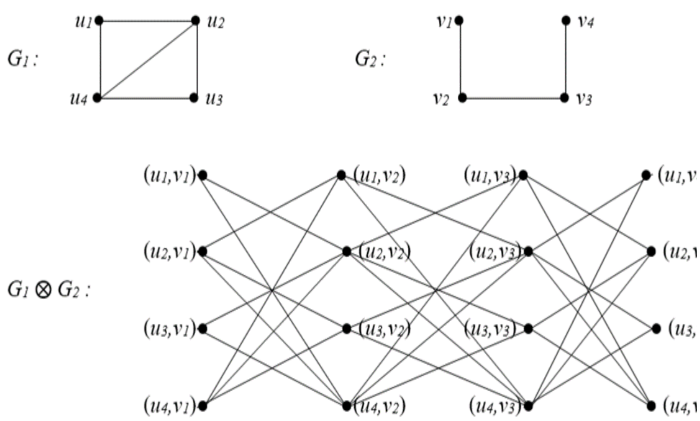
Here we define the 2-domination polynomial and recall some properties from the past works.

2.1 Definition

Consider graph G without secluded vertices. Let $\mathfrak{D}_2(G,i)$ be a group of 2-dominating sets of G with cardinality i and let $d_2(G,i) = |\mathfrak{D}_2(G,i)|$. Then the 2-domination polynomial $D_2(G,x)$ of G is explained as $D_2(G,x) = \sum_{i=\gamma_2(G)}^{|V(G)|} d_2(G,i)x^i$,where $\gamma_2(G)$ is the 2-domination number of G.

2.2 Tensor Product of Graphs

Consider $G_1 = (V_{G_1}, E_{G_1})$ and $G_2 = (V_{G_2}, E_{G_2})$ be two simple graphs. The tensor product of G_1 and G_2, represented by $G_1 \otimes G_2$, is a graph with vertex set $V_{G_1} \times V_{G_2}$ and two vertices $u = (u_1, v_1)$, $v = (u_2, v_2)$ are said to be adjoining if u_1 is adjoining to u_2 in G_1 and v_1 is adjoining to v_2 in G_2 . That is, $G_1 \otimes G_2 = (V_{G_1} \times V_{G_2}, E_{G_1} \otimes E_{G_2})$ where $E_{G_1} \otimes E_{G_2} = \{ uv \ / \ u_1 u_2 \in E_{G_1} \ and \ v_1 v_2 \in E_{G_2}\}$



3. 2-Domination Polynomial of Path

Lemma 3.1 If a graph G comprises of two parts G_1, G_2. Then

$$D_2(G,x) = D_2(G_1,x) \,.\, D_2(G_2,x)$$

Lemma 3.2 Consider a path P_n be the path having n vertices, thendomination-2 number of P_n is $\Upsilon_2(P_n) = \lceil\frac{n+1}{2}\rceil$.

Lemma 3.3 **"**Let $\mathfrak{D}_2(P_n,i)$ be a family of 2-dominating sets of P_n with cardinality i and let $d_2(P_n,i) = |\mathfrak{D}_2(P_n,i)|$. Then, $d_2(P_n,i) = d_2(P_{n-1},i-1) + d_2(P_{n-2},i-1), i \geq \lceil\frac{n+1}{2}\rceil$." [6]

Lemma 3.4 **"**Let $P_n, n \geq 3$ be a path having n number of vertices."

(i) "$d_2(P_n,i) = \phi$ if $i < \Upsilon_2(P_n)$ or $i > n$ "

(ii) "$D_2(P_n,x)s$ has no constant term and first degree terms."

(iii) "$D_2(P_n,x)$ is a strictly increasing function on $[0,\infty)$." [6]

Theorem 3.4 For every $n \geq 5$, $D_2(P_n,x) = x[D_2(P_{n-1},x) + D_2(P_{n-2},x)]$ with the initial values $D_2(P_2,x) = x^2$ and $D_2(P_3,x) = x^2 + x^3$.

4. 2 - Distance Domination Polynomial of Tensor product of P_2 and P_n

Lemma 4.1 Let P_n be the path with n vertices, then 2-domination number of $P_2 \otimes P_n$ is $\Upsilon_2(P_2 \otimes P_n) = \left\lceil\frac{n+1}{2}\right\rceil + \left\lceil\frac{n+1}{2}\right\rceil$.

Lemma 4.2 Let P_n, $n \geq 3$ be a path having n number of vertices.

$d_2(P_2 \otimes P_n, i) = \phi$ if $i < \Upsilon_2(P_2 \otimes P_n)$ or $i > 2n$.

1. $D_2(P_2 \otimes P_n, x)$ has no constants,1st degree terms, 2nd degree terms and 3rd degree terms.
2. $D_2(P_2 \otimes P_n, x)$ is a surely increasing function on $[0,\infty)$

4.3 2-Distan Domination Polynomial of Tensor product of P_2 and P_n

Theorem 4.3
For every $n \geq 5$, $D_2(P_2 \otimes P_n, x) = x^2[D_2^2(P_{n-1},x) + 2D_2(P_{n-2},x) + D_2^2(P_{n-2},x)]$ with the initial values $D_2(P_2,x) = x^2$ and $D_2(P_3,x) = x^2 + x^3$.

Proof: Let $\mathfrak{D}_2(P_2 \otimes P_n, i)$ be group of 2-dominating sets of $P_2 \otimes P_n$ having cardinality i and consider

$d_2(P_2 \otimes P_n, i) = |\mathfrak{D}_2(P_2 \otimes P_n, i)|$. Then the domination-2 polynomial $D_2(P_2 \otimes P_n, x)$ of $P_2 \otimes P_n$ is specified as $D_2(P_2 \otimes P_n, x) = \sum_{i=\gamma_2(P_2 \otimes P_n)}^{2n} d_2(P_2 \otimes P_n, i)x^i$,where $\gamma_2(P_2 \otimes P_n)$ is the domination-2 number of $P_2 \otimes P_n$. The domination-2 polynomial of the graph P_n is $D_2(P_n, x) = x[D_2(P_{n-1}, x) + D_2(P_{n-2}, x)]$ for every $n \geq 5$, with the initial values $D_2(P_2, x) = x^2$ and $D_2(P_3, x) = x^2 + x^3$. The tensor product of P_2 and P_n consists two components P_2 and P_n. So, the 2-domination polynomial of the tensor product of P_2 and P_n is the product of the 2-domination polynomials of $D_2(P_n, x)$ and $D_2(P_n, x)$. Therefore, the minimal 2-dominating set of $P_2 \otimes P_2$ consist of only one 2-dominating set with four vertices. Hence $D_2(P_2 \otimes P_2, x) = D_2(P_2, x). D_2(P_2, x) = x^4$.

Similarly,$D_2(P_2 \otimes P_3, x) = D_2(P_3, x). D_2(P_3, x) = (x^2 + x^3)(x^2 + x^3) = x^4 + 2x^5 + x^6$ And $D_2(P_2 \otimes P_4, x) = D_2(P_4, x). D_2(P_4, x) = (2x^3 + x^4)(2x^3 + x^4) = 4x^6 + 4x^7 + x^8$.

Hence, $D_2(P_2 \otimes P_n, x) = D_2(P_n, x). D_2(P_n, x)$

$$= x[D_2(P_{n-1}, x) + D_2(P_{n-2}, x)]x[D_2(P_{n-1}, x) + D_2(P_{n-2}, x)]$$

$$= x^2[D_2^2(P_{n-1}, x) + 2D_2(P_{n-2}, x) + D_2^2(P_{n-2}, x)]$$

5. Conclusion and Future Enhancements

In this article, we have derived the domination-2 polynomials of tensor product of paths P_2 and P_n from its 2-dominating sets of P_n. Presently, we are working on the 2-domination polynomials of tensor product of paths P_n and P_m.

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