UOIuBIH
ORSinBIH

Operations Research Society in
Bosnia and Herzegovina

Southeast Europe Journal of Soft Computing

Available online: www.scjournal.com.ba

Real Time Distribution System State Estimation Based On Interior Point Method

Izudin Dzafic, Indira Huseinagic, S. Henselmeyer

International University of Sarajevo, Faculty of Engineering and Natural Sciences, Hrasnicka Cesta 15, Ilidža
71210 Sarajevo, Bosnia and Herzegovina

Article Info

Article history:

Article received on January 2014

Keywords:

Power distribution systems, distribution system state estimation, load modeling, weighted least absolute value, equality constraints.

Abstract

This approach for a three-phase Distribution System State Estimator (DSSE) considers all the analog measurements including voltage and current measurements, active and reactive power measurements, as well as historical load-information. The estimation of load information is based on an interior point optimization. It reduces the estimation problem to load groups which are located in so called measurement areas. By grouping the loads with the same weighting factors, the number of state variables for the optimization is minimized. The power mismatches at the boundary between neighboring areas are eliminated by equality constraints. This significantly reduces size of the estimation problem. The proposal is designed for radial as well as for networks with some meshes.

NOMENCLATURE

NLG	total number of load groups.
$NLKG$	number of load groups in area k.
NPQ	total number of power measurements in system including the ones converted from currents.
NI	total number of current measurements.
NMK	number of P-/Q- pairs converted from currents in area k.
NGK	number of generators in area k.
NCK	number of capacitors in area k.
$NVNA$	number of virtual node area.
$NLDk$	number of loads connected at virtual node areas k.
$NBRk$	number of branches connected at virtual node areas k.
P, Q_i^{LG}	active and reactive power of i-th load group.
P, Q_i^E	estimated active and react. power of i-th measurement.
P, Q_i^m	measured active and reactive power including pseudo measurements converted from currents.

P, Q_i^{gen}	active and reactive power of the i-th generator.
$\Delta P, Q$	sum of the losses in the measurement area, the measured values at the boundaries and all internal generation.
$w_i^{P, LG}$	weighting factors for active power of i-th load group.
$w_i^{Q, LG}$	weighting factors for reactive power of i-th load group.
w_i^{IM}	weighting factor for current measurements.
$I_i^{E, M}$	estimated and measured i-th current magnitude.
$V_i^{E, M}$	estimated and measured i-th voltage.
$P, Q_i^{E, k}$	estimated load i connected at virtual node areas k.
P, Q_i^{EBrk}	estimated flow on branch i connected at virtual node area k.
I_M	measured current at the boundary branch.
P, Q_{loss}^k	power losses in area k.
P, Q_C^E	estimation results for a current converted to power.
I_C^E	estimation results for a current converted to power.

- $V, \varphi_i^{E,V,N}$ voltage and angle at virtual node area estimated by WLAV algorithm.
- $V, \varphi_i^{c,V,N}$ voltage at virtual node area calculated by power flow.
- $k_{pi}^{min,max}$ lower and upper limits for P scaling factor of a load group.
- $k_{qi}^{min,max}$ lower and upper limits for Q scaling factor of a load group.

1. INTRODUCTION

From last several decades state estimation has been successfully established for the calculation of transmission systems. Since [1] proposed the estimation of the state variables of energy system in 1970, state estimation has been intensively improved. Using a redundant set of active and reactive power, current and voltage measurements, it has become possible to calculate observable parts. As long as the local redundancy is sufficient, even large measurement errors can be identified ([3], [4], [5]). However, the advantages of the approaches used in transmission systems cannot be directly transferred to distribution systems, due to the completely different electrical properties in distribution systems such as: radial or tree-like topology, large R/X ratio for cables, small number of real-time measurements, mostly current- instead of power measurements, unbalanced loading, unsymmetrical construction. The missing redundancy of real time measurements is a more serious problem. Thus, different models for distribution systems are required. This problem has been addressed in many publications. For the optimal placing of measurements in a network in order to get it observable, several methods are proposed ([6]- [9]). For the loads of the network, additional assumptions are required. Generally, their time behavior is defined by historical data, e.g. by load curves, or with typifying loads, which have a similar behavior. But, in [17] it is pointed out, that this historical information does normally not match the real time, and additional treatment is required for the load curve values. In [15], [16], a load flow approach is proposed, where the loads are scaled in such a way, that the loads and measurements fit. This approach has some disadvantages, especially, when there are some loops in the network. Also, the use of current measurements is not completely satisfying. Approach is based on estimation of the system's state, using a WLS-estimator ([10], [11], [20]-[22]). It is mainly designed for radial systems and mostly based on branch currents as state variables. In this paper, a new algorithm is presented for an improved estimation of the loads. The optimization problem can be significantly reduced by defining so called measurement areas, which are characterized by measurements on the boundary branches to the neighboring measurement areas. For each of areas, loads groups are defined and then adapted by an estimation of scaling factors. A power mismatch at the boundaries of these areas is eliminated by equality constraints. Thus, this new approach is a combination of a load flow based scaling and an interior point optimization. It is also

applicable for the calculation of distribution networks with some meshes.

2. DESCRIPTION OF THE IDEA

Typically, the structure of a distribution system is based on radial or tree like feeders, probably with loops, which are often operated as open rings. In these networks, measurements are mainly located at the feeder-heads, either as voltage measurements, active and reactive power measurements or as current measurements.

Figure 1. Small distribution feeder with tree like topology, open ring and some measurements areas (MA)

Figure1 represents a small network with an openly operated loop. Both radial feeders have some measurements of different types. Using these types, sets of measurement areas are defined. The load information in these measurement areas is to be scaled in such a way, that the network areas are balanced with respect to the injected power and the network losses, the measured information is matched, the power losses of these areas are correctly considered and power mismatch at the boundary branches between two measurement areas is eliminated. The neighboring measurement areas are coupled via the flows between these areas (Figure2).

Figure 2. Equivalent injections at the boundaries between different measurement areas (feeder 1 of Fig.1)

The flow leaving one area is injected to another. This is used for an improved scaling based on weighted least absolute value estimation. With these couplings between neighboring measurement areas, an iterative process can be started with

- a load flow calculation for the complete network
- an estimation which adapts the loads and eliminates the power mismatch at the boundaries.

When using different weighting for different quality of information, the load information as well as measured information or pseudo measurements can be treated in the same way. This ensures, that the real measurements are matched (large weighting), whereas pseudo measurements or load curve values allow a larger discrepancy between the input and the calculated value (low weighting). The optimization uses the load information as state variable. The structure and the admittances of measurement areas are not directly required; it only needs the sum of their scaled loads in the different iteration steps, the losses and the loading of the boundary branches. Voltages and currents are indirectly used after a power flow calculation has been executed. Defining groups of loads which have the same weighting reduces the effort for the estimation significantly in each iteration step. Such a load group consists of loads, which have the same quality of information, thus, they are assigned the same weighting. Consequently, their values are scaled with the same factors. In figure 3 the flow chart of the proposal is displayed.

Figure 3. Flow chart of DSSE algorithm

3. CONVERSION OF CURRENT MEASUREMENTS

The scaling process is pretty simple for areas, which have complete sets of P - and Q - measurements. The loads are scaled in such a way, that they match the sum of the injected and measured sum of the power at the boundary. The more critical workflow is given for injections into a measurement area, which is only based on current

measurements. Current measurements cannot be directly used for the scaling of the load information, and conversion of current information to a complex power becomes necessary. For each measurement area, the conversion of current measurements to P - and Q - pseudo measurements is executed individually. For each area with only one current measurement at its boundary, following objective is minimized

$$\sum_i^{NLG} \left(w_i^{P, LG} \left| P_i^{LG} (k_i^P - 1) \right| + w_i^{Q, LG} \left| Q_i^{LG} (k_i^Q - 1) \right| \right) \quad (1)$$

using the constraints

$$\left(\sum_{i=1}^{NLG} k_i^P P_i^{LG} + \Delta P \right)^2 + \left(\sum_{i=1}^{NLG} k_i^Q Q_i^{LG} + \Delta Q \right)^2 = V^2 I_M^2 \quad (2)$$

$$k_{pi}^{\min} \leq k_i^P \leq k_{pi}^{\max}$$

$$k_{qi}^{\min} \leq k_i^Q \leq k_{qi}^{\max}$$

In the next estimation step, the scaled values k_i^P, P_i^{LG} and k_i^Q, Q_i^{LG} are used as new starting point. The conversion of current measurements starts with the measurement areas where is only one current measurement to be converted. It is executed “bottom up”, starting with the area which only has one current measurement which was not converted yet. For feeder 2 in figure1, measurements area 6 is considered after 7 and 8, because it has 2 branches with current measurements. Converting the current measurements of area 7 and 8, the number of the current measurements in area 6, which are not converted, is reduced. On this way, the number of non converted current measurements is reduced until all of them are converted.

4. STATE ESTIMATION

As the conversion of currents to P - and Q -pseudo measurements is done individually for each measurement, existing couplings between these areas is not considered. They are considered in the next estimation step, which takes into account the complete network. This step forces the flow at the boundaries of neighboring areas to match. The estimation problem is defined as a minimization of the objective J described by the equation (3). The first term of equation (3) considers, that the differences between measured and estimated power information is minimized, the second term is responsible for the minimization of the differences in the load information. The third term considers the differences between measured and estimated currents. Additionally the following equality constraints must be fulfilled:

- Sum of estimated P for real and pseudo measurements and P contributed by internal generators covers active power losses (equation (4))
- Sum of estimated Q for real and pseudo measurements and Q contributed by generators and capacitors covers reactive power losses (equation(5))

- For each area with pseudo measurements converted from current measurement, the estimated P_C^E , Q_C^E , I_C^E and V_C must fulfill equation (6)

$$J = \min \left(\sum_{i=1}^{NPQ} \left[w_i^{PM} |P_i^E - P_i^m| + w_i^{QM} |Q_i^E - Q_i^m| \right] + \sum_{i=1}^{NLG} \left[w_i^{PLG} |P_i^{LG} (k_i^P - 1)| + w_i^{QLG} |Q_i^{LG} (k_i^Q - 1)| \right] + \sum_{i=1}^{NI} w_i^{IM} |I_i^E - I_i^m| \right) \quad (3)$$

with the equality constraints

$$\sum_{i=1}^{NMK} P_i^E + \sum_{i=1}^{NGK} P_i^{Gen} - \sum_{i=1}^{NLGK} k_i^P P_i^{LG} - P_{loss}^k = 0 \quad (4)$$

$$\sum_{i=1}^{NMK} Q_i^E + \sum_{i=1}^{NGK} Q_i^{Gen} + \sum_{i=1}^{NCK} Q_i^{Cap} - \sum_{i=1}^{NLGK} k_i^Q Q_i^{LG} - Q_{loss}^k = 0 \quad (5)$$

$$\left(P_C^E \right)^2 + \left(Q_C^E \right)^2 - \left(V_C I_C^E \right)^2 = 0 \quad (6)$$

and the inequality constraints same as in equation (2).

5. STATE ESTIMATION WITH INTERIOR POINT OPTIMIZATION

An introduction in interior point methods is given in [22]. The estimation approach, described here, is mostly based on this presentation. The problem for both current to power conversion and the estimation for the whole system can be defined as the following minimization problem:

$$f(\mathbf{x}) = \sum_i^n w_i |x_i - x_i^M| \quad (7)$$

subject to

$$g(x) = 0 \\ \underline{h} \leq h(x) \leq \bar{h} \quad (8)$$

where x_i , x_i^M are i-th component of a n dimensional vector of measurement and estimated state, w_i is i-th component of a n dimensional vector of weighting factors, $g(x)$, $h(x)$ are m dimensional vector of equality and inequality constraints, and $\bar{h}$, $\underline{h}$ p dimensional vectors with lower and upper limits for inequality constraints. The given optimization problem is transformed while replacing inequality constraints by corresponding equality constraints using non-negative slack vectors:

$$-s - z + \bar{h} - \underline{h} = 0 \\ -h(x) - z + \bar{h} = 0 \quad (9)$$

where s, z are p dimensional vectors of slack variables, and $s, z \geq 0$. The non-negativity conditions are satisfied while introducing logarithmic barrier terms in the problem formulation:

$$\min \sum_i^n w_i |x_i^M - x_i| - \mu^k \sum_i^p (\ln s_i + \ln z_i) \quad (10)$$

where μ^k is barrier parameter with $\mu^k > 0$, decreasing to zero. The entire problem is solved using Lagrange multipliers combined with Merhotra's Predictor-Corrector Method:

$$L_\mu(y) = \sum_i^n w_i |x_i^M - x_i| - \mu^k \sum_i^p (\ln s_i + \ln z_i) - \lambda^T g(x) - \pi^T (-s - z + \bar{h} - \underline{h}) - v^T (-h(x) - z + \bar{h}) \quad (10)$$

where $y = (s, z, \pi, v, x, \lambda)^T$ and π, v is p dimensional vectors of Lagrange multipliers. The Mehrotra's Predictor-Corrector Method reduces the number of iterations until convergence while including additional information into standard Newton representation. The third term of the right hand side of (11) is the extended part. It is based on the incorporation of the new point $y^{k+1} = y^k + \Delta y$ into Newton representation as shown in [22].

$$\nabla_y^2 L_\mu(y) \Delta y = \begin{pmatrix} -S\pi \\ -Z\hat{v} \\ r_\pi \\ r_v \\ r_x \\ r_\lambda \end{pmatrix} + \begin{pmatrix} \mu^k e \\ \mu^k e \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \begin{pmatrix} -\Delta S \Delta \pi \\ -\Delta Z \Delta \hat{v} \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \quad (11)$$

where

$$r_v = -h(x) - z + \bar{h}, \\ r_x = -\nabla_x f(x) + J_g(x)^T \lambda - J_h(x)^T v \\ r_\lambda = g(x)$$

S is a p by p matrix with vector s at the diagonal, Z is a p by p matrix with vector z at the diagonal, e is identity matrix, $\hat{v}$ is sum of vectors v and π . The desired solution is split into a predictor- and a corrector- step. In the predictor-step, the system is solved for the pure Newton direction (also called affine-scaling direction), i.e. without barrier parameter and delta terms in the right hand side of (11) [22]. The found solution is used to estimate the barrier parameter dynamically and to approximate delta terms from right hand side. For updating the update barrier parameter, the step length parameters are first calculated for the pure Newton direction:

$$\alpha_P^{af} = \min \left\{ 1, \gamma \min_i \left\{ \left| \frac{-s_i^k}{\Delta s_i^{af}} \right|_{\Delta s_i^{af} < 0}, \left| \frac{-z_i^k}{\Delta z_i^{af}} \right|_{\Delta z_i^{af} < 0} \right\} \right\} \\ \alpha_D^{af} = \min \left\{ 1, \gamma \min_i \left\{ \left| \frac{-\pi_i^k}{\Delta \pi_i^{af}} \right|_{\Delta \pi_i^{af} < 0}, \left| \frac{-\hat{v}_i^k}{\Delta \hat{v}_i^{af}} \right|_{\Delta \hat{v}_i^{af} < 0} \right\} \right\} \quad (12) \\ \alpha_P^{af} = \alpha_D^{af} \leftarrow \min \{ \alpha_P^{af}, \alpha_D^{af} \}, \quad \alpha_P^{af}, \alpha_D^{af} \in (0,1]$$

where

$\alpha_p^{af}, \alpha_D^{af}$	step length parameters
$s_i^k, z_i^k, \pi_i^k, \hat{v}_i^k$	solution of the k-th iteration
$\Delta s_i^{af}, \Delta z_i^{af}, \Delta \pi_i^{af}, \Delta \hat{v}_i^{af}$	delta step for affine scaling direction

The step length parameters are used to calculate the complementarity gap:

$$\rho_{af}^k = \left(s^k + \alpha_p^{af} \Delta s_{af} \right)^T \left(\pi^k + \alpha_D^{af} \Delta \pi_{af} \right)^T + \left(z^k + \alpha_p^{af} \Delta z_{af} \right)^T \left(\hat{v}^k + \alpha_D^{af} \Delta \hat{v}_{af} \right) \quad (13)$$

where ρ_{af}^k is complementarity gap for affine scaling direction. The barrier parameter is finally calculated from:

$$\mu_{af}^k = \min \left\{ \left(\frac{\rho_{af}^k}{\rho^k} \right)^2, 0.2 \right\} \frac{\rho_{af}^k}{2p} \quad (14)$$

Where $\rho^k = (s^k)^T \pi^k + (z^k)^T \hat{v}^k$. In the corrector step, the full Newton direction Δy is calculated. Missing barrier parameter value and delta terms are approximated using results from previous predictor step. Both predictor and corrector steps use the same matrix factorization.

$$\begin{bmatrix} W & G^T \\ G & 0 \end{bmatrix} \begin{bmatrix} \Delta x \\ -\lambda \end{bmatrix} = \begin{bmatrix} W \Delta \hat{x}^k \\ -g(x^k) \end{bmatrix} \quad (15)$$

where

x^k	solution of the k-th iteration
x^M	vector of measurements including pseudo measurements
$\Delta x = x^{k+1} - x^k$	vector with differences between results of two subsequent iterations
$\Delta \hat{x}^k = x^M - x^k$	vector with differences between results of the k-th iteration and the measurements
$G = \frac{\partial g(x)}{\partial x}$	matrix with the partially derivatives of the constraints vector.

6. HANDLING OF MESHED TOPOLOGY

When there are two or more current measurements on boundary branches to one measurement area, the conversion of current measurements to pseudo power flow information is not uniquely possible. In many cases, this problem is simply solved by converting all the current measurements leading to an area with only one non-converted current measurement exists. The number of non-converted currents is decreased for the neighboring area. This is always successful for radial or tree-like networks. In general, this is not given for networks with some meshes or a parallel injection. The impedances inside the loop or the coupling branches are to be considered in the estimation process. Each branch of such an area with

several remaining current information is defined to be a measurement area of its own, having a defined active and reactive virtual information at each of its terminals. These measurements are called virtual, because they are not derived from a real current measurement. The complex power at both sides of a branch, which leads from i to j , is given by

$$\begin{bmatrix} S_{ij} \\ S_{ji} \end{bmatrix} = \begin{bmatrix} V_i & 0 \\ 0 & V_j \end{bmatrix} \begin{bmatrix} I_i \\ I_j \end{bmatrix}^* = \begin{bmatrix} V_i & 0 \\ 0 & V_j \end{bmatrix} \left(\begin{bmatrix} Y_{ii} & Y_{ij} \\ Y_{ji} & Y_{jj} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix} \right)^* \quad (16)$$

From the preceding load flow calculation, this power information is extracted and used like low weighted measured information. Please note, that the calculation of these virtual power measurements is only necessary for the sides of the branches which is connected to a node inside the meshed network part. No calculation is executed for branch sides, where real power measurements are available. Fig. 5 display the newly created measurement areas and the corresponding virtual measurements. In following, the newly created areas, which contain no branches, are called virtual node areas.

Figure 5. Subdivided measurement areas inside a meshed network.

The objective in equation (3)-(6), which are used for the optimization process, is to be extended by the additional virtual information according to

$$J' = \min \left(J + \sum_{i=1}^{NVNA} \left(w_{V_i}^{NVNA} \left| V_i^{E,VN} - V_i^{c,VN} \right| \right) + \sum_{i=1}^{NVNA} \left(w_{\phi_i}^{NVNA} \left| \phi_i^{E,VN} - \phi_i^{c,VN} \right| \right) \right) \quad (17)$$

The virtual node areas must fulfill the first Kirchhoff -law.

$$\sum_{i=1}^{NLDk} P_i^{Ek} + \sum_{i=1}^{NBRk} P_i^{E,Brk} = 0 \quad (18)$$

$$\sum_{i=1}^{NLDk} Q_i^{Ek} + \sum_{i=1}^{NBRk} Q_i^{E,Brk} = 0 \quad (19)$$

The virtual power measurements are considered as real power measurements during estimation, but with a low weighting. They have their origin in the load flow calculation, which precedes the estimation.

6. TEST CASES

In the following, test cases for a modified IEEE34 network [24] are discussed. Fig. 6 shows the schema for this network. The network is extended by an injection transformer and additional buses. There are 6 P - and Q -measurement pairs at injection transformer and at the feeder head for each phase and 19 current magnitude measurements downstream the feeder head.

Figure 6. Modified IEEE34 network with used measurements (marked by “X”).

The weighting factors for all the real measurements are equally set to 900, the weighting factors for load data are set to 0.5. The test scenario mirrors the situation in a distribution system: low number of real measurements; more current measurements available than power measurements; real measurements are more reliable than pseudo measurements at loads. Measurement values are obtained from power flow engine while the load input data are multiplied with a factor of 1.6. The inequality constraints for load scaling factors are derived from load setup and set to the range [1,2]. That means, that calculated load scaling factors should be less than 2, but higher than 1.

Table1: test results for measurements in modified ieee34.

Branch	Node	Type	Deviation [%]		
			A	B	C
-	799	V [Kv]	0	0	0
799-800	799	P [MW]	0.32	0.32	1.89
799-800	799	Q[MVAr]	0	0.46	0.49
800-802	800	I [A]	0.37	0.35	1.77
800-802	800	P [MW]	0.45	0.41	2.38
800-802	800	Q[MVAr]	0.19	0.22	0.25
816-850	816	I [A]	1.41	0.37	2.38
816-818	816	I [A]	0.29		
816-824	816	I [A]	2.09	0.32	2.38
834-858	834	I [A]	0.43	0.5	0.52
834-842	834	I [A]	0	0.33	0.76
834-860	834	I [A]	0.43	0.42	0.4

Table1 summarizes the results for given real measurements. The deviation between measured and calculated data is below 0.5%, the maximum is 2.38% for some measurements at phase C. The average deviation is 0.67%. Compared with the results in [25], where a WLS minimization was used, the deviations between the input information and the calculated results are significantly reduced by use of interior point optimization. This approach delivers more accurate results.

Table 2: test results for loads in modified ieee34.

Node	Phase	Type	Scaling Factor P-Q		
			A	B	C
802-806	BC	Y	0-0	1.59-1.76	1.66-1.76
808-810	B	Y	0-0	1.62-1.79	0-0
818-820	A	Y	1.84-1.87	0-0	0-0
820-822	A	Y	1.47-1.66	0-0	0-0
816-824	BC	Delta	0-0	1.24-1.33	0-0
824-826	B	Y	0-0	1.24-1.34	0-0
824-828	C	Y	0-0	0-0	1.65-1.6
828-830	A	Y	1.19-1.8	0-0	0-0
830	ABC	Delta	1.39-1.5	1.37-1.48	1.39-1.49
854-856	B	Y	0-0	1.15-1.24	0-0
858-832	ABC	Delta	1.56-1.69	1.58-1.7	1.55-1.67
858-864	ABC	Y	1.19-2.04	1.15-1.24	1.65-1.6
858-834	ABC	Y	1.19-2.04	1.15-1.24	1.65-1.6
842-844	A	Y	1.62-1.89	0-0	0-0
844-846	BC	Y	0-0	1.59-1.61	1.64-1.43
846-848	B	Y	0-0	1.59-1.61	0-0
848	ABC	Y	1.62-1.49	1.59-1.61	1.64-1.43
834-860	ABC	Y	1.87-1.87	1.98-1.98	1.87-1.81
860	ABC	Y	1.38-1.38	1.39-1.42	1.4-1.35
836-860	ABC	Y	1.38-1.38	1.39-1.42	1.4-1.35
836-840	ABC	Y	1.61-1.61	1.66-1.7	1.62-1.57
840	ABC	Y	1.61-1.61	1.66-1.7	1.62-1.57
838-862	B	Y	0-0	1.39-1.42	0-0
890	ABC	Delta	1.58-1.7	1.43-1.54	1.58-1.7

Table 2 shows the deviation between input data and results for loads. Load scaling factors, which express the ratio between estimated and original load data, are in a range [1.24, 1.98]. As the measurements have been multiplied with a factor of 1.6 for this calculation, this is an excellent result. All the loads were scaled by some factor around this multiplier. As expected the mismatch between real measurements and load data is shifted mainly to load data which have a poor accuracy.

6. CONCLUSION

This paper presents a new approach for a distribution system state estimation. This approach is a combination of power flow based load scaling and a WLAV-state estimation with equality and inequality constraints. For the estimation process itself, the set of state variables are sets of load information, which are used as input for a load flow calculation. With different weightings to the available information, this approach allows to model the accuracy or the available information. This and the use of inequality constraints increase the accuracy of the results. Further investigations are intended with respect to the detection and identification of errors in the measurements and load data.

REFERENCES

- [1] F. C. Schweppe and J. Wildes, "Power system static-state estimation, part I, II, III," IEEE Trans. Power App. Syst., vol. PAS-89, no. 1, pp. 120–135, Jan. 1970.
- [2] F. F. Wu, "Power System State Estimation: A Survey," Electrical Power & Energy Systems, Vol. 12, No. 2, April 1990.
- [3] Wu, F.F.; Liu, W.-H.E.; Holten, L.; Gjelsvik, L.; Aam, S.; "Observability analysis and bad data processing for state estimation using Hachtel's augmented matrix method", IEEE Transactions on Power Systems, Vol. 3, 1988 , pp 604 - 611
- [4] Alves da Silva, A.P.; Quintana, V.H.; Pang, G.K.H.; "A pattern analysis approach for topology determination, bad data correction and missing measurement estimation in power systems", Proceedings of the Twenty-Second Annual North American Power Symposium, 1990, pp 363 - 372
- [5] Beissler, G.; Koglin, H.-J.; Neisius H.-T.; Schmitt, K: "Bad Data Detection and Identification." Eps. Vol 12, no. 2, 1990, pp. 94-103
- [6] B. Gou and A. Abur, "An improved measurement placement algorithm for network observability," IEEE Trans. Power Syst., vol. 16, no. 4, pp. 819–824, Nov. 2001.
- [7] Baran M. E., J. Zhu, A. W. Kelley, "Meter Placement for Real-Time Monitoring of Distribution Feeders," IEEE Trans. on Power Systems, Feb. 1996, pp. 332-337.
- [8] K. A. Clement, G. R. Krumpholz, P. W. Davis, "Power system state estimation with measurement deficiency: an observability measurement placement algorithm," IEEE Trans. on Power Apparatus and Systems, Vol. PAS-102, July, 1983, pp. 2012-2020.
- [9] A. Monticelli and F. F. Wu, "Network Observability: Identification of Observable Islands and Measurement Placement," IEEE Trans. on Power Apparatus and Systems, Vol. PAS-104, May 1985, pp. 1035-1041
- [10] C. N. Lu, J. H. Teng, and W. H. E. Liu, "Distribution system state estimation," IEEE Trans. Power Syst., vol. 10, pp. 229–240, Feb. 1995.
- [11] H. Wang, and N. N. Schulz, "A Revised Branch Current-Based Distribution System State Estimation Algorithm and Meter Placement Impact", IEEE Transactions on Power Systems, Vol. 19, No. 1, February 2004, pp 207-213.
- [12] R. Singh, B. C. Pal, and R. B. Vinter, "Measurement Placement in Distribution System State Estimation", IEEE Transactions on Power Systems, Vol. 24, No. 2, May 2009, pp 668-675.
- [13] K. A. Clement, G. R. Krumpholz, and P. W. Davis, "State Estimator Measurement System Reliability Evaluation - An Efficient Algorithm Based on Topological Observability Theory," IEEE Trans. on Power Apparatus and Systems, April 1982, pp. 997-1004.
- [14] K. A. Clements and P. W. Davis, "Detection and identification of topology errors in electric power systems," IEEE Trans. Power Syst., vol. 3, no. 4, pp. 1748–1753, Nov. 1988.
- [15] I. Roytelman, and S. M. Shahidehpour, "State Estimation for Electric Power Distribution Systems in Quasi Real-Time Conditions", IEEE Transactions on Power Delivery, Vol. 8, No. 4, October 1993, pp 2009-2015.
- [16] Lubkeman, D.L.; Jianzhong Zhang; Ghosh, A.K.; Jones, R.H, "Field results for a distribution circuit state estimator implementation", IEEE Trans. on Power Delivery, Jan. 2000, pp. 399-406
- [17] E. Handschin M. Lamger, E. Kliokys: "An Interior Point Method For State Estimation With Current Magnitude Measurements And Inequality Constraints", PICA, Conference Proceedings. 1995 IEEE, pp. 385 - 391
- [18] H. Wang and N. N. Schulz, "A load modeling algorithm for distribution system state estimation," in Proc. 2001 IEEE/Power Eng. Soc. Transm. Dist. Conf. Expo., pp. 102–106.
- [19] Atish K. Ghosh, David L. Lubkeman, Robert H. Jones: "Load modeling for distribution circuit state estimation" IEEE Transactions on Power Delivery, Vol. 12, No. 2, April 1997, pp 999 1005
- [20] Baran, M.E., and A. W. Kelley, "State Estimation for Real-Time Monitoring of Distribution Systems," IEEE Trans. on Power Systems, Aug. 1994, pp. 1601-1608,
- [21] Baran, M. McDermott, T.E., "Distribution System State Estimation Using AMI Data", Power Systems Conference and Exposition, 2006. PSCE '06. 2006 IEEE PES
- [22] Task Force on "Interior-Point Method Applications to Power Systems", IEEE-PES Mini Lecture, Proceedings of PICA'99, Santa Clara, CA, 1999.
- [23] W. H. Kersting, "Distribution System Modeling and Analysis", CRS Press 2007
- [24] <http://ewh.ieee.org/soc/pes/dsacom/testfeeders/index.html>
- [25] I. Dzafic, S. Henselmeyer, H.T. Neisius, "Real-Time Distribution System State Estimation", IPEC, Singapur, October 2010.