UOIuBIH
ORSinBIH
Operations Research Society in Bosnia and Herzegovina

Southeast Europe Journal of Soft Computing
Available online: http://scjournal.ius.edu.ba

IUS Soft Computing Research Group

Optimal Combination of Three Volatilities for Better Black-Scholes Option Pricing

S. Fadda
International University of Sarajevo
Faculty of Business Administration
Hrasnicka Cesta Ilıdza, Sarajevo BiH 71210
sadifadda@gmail.com

Article Info

Article history:
Article received on 10 January 2018
Received in revised form 17 February 2018

Keywords:
Black-Scholes, Implied volatility, GARCH (1,1), GJR(1,1), optimal combination, option pricing, S&P100, put options, call options.

ABSTRACT: The most popular parametric formula (B-S) used in pricing the European-style options is given by Black-Scholes (1973). The prediction power of (B-S) strongly rely on the accuracy of the independent variables: spot price, strike price, time to maturity, risk free interest rate and market volatility. To improve the accuracy, many volatility models are proposed. Also Day and Lewis (1992) introduced the idea of combining implied volatility and EGARCH. In this article an optimal combination of market implied volatility, GARCH(1,1), and GJR(1,1) is made, and the prediction power of B-S is doubled.

I. INTRODUCTION

The main measure of an investment risk, whether of a single financial instrument stock, bond, forward or a portfolio, is its volatility. While considering the simplicity of calculating an implied volatility based on a given market price of an instrument, it is the volatility of the approaching period through which the instrument is to be held that actually matters. In this paper, three different volatilities are combined to get a new one.

For many decades Black-Scholes (Black, and Scholes 1973) formula and its modifications are used to price European style put, and call options. The formula depends on five parameters, spot price, *S*, strike price, *K*, time to maturity, *T*, risk free interest rate *r*, and market volatility s. For calls "c" or puts "p" the formulae are

$$c = S\,N(d_1) - Ke^{-rT}\,N(d_2), \tag{1}$$

and

$$p = Ke^{-rT}\,N(-d_2) - SN(-d_1) \tag{2}$$

Where

$$d_1 = \frac{\ln(S/K) + (r + s^2/2)T}{s\sqrt{T}}, \tag{3}$$

and

$$d_2 = d_1 - s\sqrt{T}. \tag{4}$$

In (1) and (2), N(x) is the cumulative standard normal distribution function. Indeed the probability that option gets exercised is $N(d_2)$. It is multiplied by the strike price and then discounted to its present time value, giving the expected value of the cost of exercising an option.

N(d1) is the factor which measures how much the present value of the asset exceeds its current market price (Nielsen 1992). The product of N(d1) and option spot-price is the expected value of receiving the stock at option's maturity. Therefore the option price must be equal to the difference of these two expected values.

There are many objections to B-S option pricing formula. Fisher Black (Black 1975) states that the market prices of listed options tend to differ from the values calculated by the Black-Scholes formula methodically. Especially extremely-out-of-money options tend to be overpriced, while into-the-money ones tend to be underpriced. He also adds that options having time-to-maturity below three months do tend to be overpriced.

Since the introduction of Black-Scholes-Merton in (Black, and Scholes 1973), most of the research efforts have been to remove some of the restrictions or assumptions in the Black–Scholes model. Among those are the premises of the log-normality of returns or that of constant volatility.

Meanwhile several volatility models are proposed. Implied, historical, conditional probabilities are some of them (Fadda, 1916; Fadda, and Can 1917). Works (Latané and Rendleman 1976; Schmalensee, and. Trippi 1978; Chiras, and Manaster 1978), supported the claim that implied volatility outperforms historical volatility. In (Beckers, 1981) it is demonstrated that the implied volatilities of at-the-money options give better results than any other combination of available implied volatilities.

In this article, using the implied volatility computed from B-S formula as the target, and a least square formulation, an optimal combination of the three volatilities, implied volatility, GARCH(1,1), and GJR(1,1), the prediction power of B-S is doubled.

2. MATERIALS AND METHODS

The data used in this article is S&P100 European Style Index also known as XEO, 75, 694 call options throughout one year from October3, 2013 to September 29, 2014, and 141,397 put options in the same period.

To find the optimum combination of the three volatility models, first a target volatility, that is when used with other parameters in B-S formula, will exactly yield the market price of the option, is computed. It is observed that the option price in B-S formula is an increasing function of the volatility. An iterative scheme is set. Iteration starts by the implied volatility supplied by the data set. If computed option price is less than the market price of the option, and then volatility is slightly increased till, computed option price is the same as the market price. If computed option price is more than the market price of the option, and then volatility is slightly decreased till, computed option price is the same as the market price. The volatility V of the balanced state is a kind of implied volatility of the B-S formula. Then a linear combination

$$V = aX + bY + cZ + d \tag{5}$$

of X= implied volatility, Y= GARCH(1,1), and Z= GJR(1,1) is found, which is the best in the sense of least squares. The coefficients of optimal combination are found as in Table 1.

Table 1. Coefficients of optimal combination for options

	a	b	c	d
Puts	0.8099	0.5858	-0.4629	0.0610
Ccalls	0.8769	0.4952	-0.3737	0.0439

Option price depends on the parameters strike price, *K*, time to maturity, *T*, risk free interest rate *r*, which are known from today. If we predict the tomorrow's spot price *S*, using the best estimate *s* for the market volatility, the optimum option prices for tomorrow can be calculated. Option prices in our dataset are predicted one day before, and when compared with the realized prices, it is seen that the success of B-S formula is doubled.

Table 2. Absolute mean errors in pricing

	Implied Volatility	Optimal Volatility
Puts	5.2490$	3.5773$
Calls	6.0252$	3.6573$

3. CONCLUSIONS

Since the introduction of Black-Scholes-Merton in 1973, most of the research efforts have been targeted to remove constant volatility condition. For this several volatility models are proposed. Implied, historical, conditional probabilities are among them. Some researchers also tried optimization processes to assign appropriate weights to volatilities involved. In this research work implied volatility, GARCH(1,1), and GJR(1,1) are combined thorough a least squares technique, and , the prediction power of B-S is doubled.

REFERENCES

Beckers, S. (1981) Standard deviations implied in option prices as predictors of future stock price variability, Journal of Banking & Finance, Volume 5, Issue 3, Pages 363-381

Black, F., and Scholes, M. (1973) The Pricing of Options and Corporate Liabilities, The Journal of Political Economy, Vol. 81, No. 3, pp. 637-654

Black, F. (1975) Fact and Fantasy in the Use of Options, Financial Analysts Journal, Volume 31 Issue 4 pp. 36-41

Chiras, D. P., and Manaster, S. (1978) The information content of option prices and a test of market efficiency, Journal of Financial Economics, Volume 6, Issues 2–3, Pages 213-234.

Day, T.E., and Lewis, C.M. (1992) Stock market volatility and the information content of stock index options Journal of Econometrics 52, 267-287. North-Holland

Fadda, S. (2016) Volatility Estimation Through Historical Prices of Indexes Southeast Europe Journal of Soft Computing Vol.5 No, (21-26)

Fadda, S., and Can, M. (2017) Forecasting Conditional Variance of S&P100 Returns Using Feedforward and Recurrent Neural Networks Southeast Europe Journal of Soft Computing Vol.6 No.1, (58-69)

Latané, H. A., and Rendleman, R. J. (1976), Standard Deviations Of Stock Price Ratios Implied In Option Prices, The Journal of Finance, Volume 31, Issue 2, Pages 369–381

Nielsen, L.T. (1992) Understanding N(d1) and N(d2): Risk-Adjusted Probabilities in the Black-Scholes Model , INSEAD, Boulevard de Constance, 77305 Fontainebleau Cedex, France.

Schmalensee, R., and Trippi, R.R. (1978) Common Stock Volatility Expectations Implied By Option Premia, Journal of Finance, Volume 33, Issue 1, Pages 129–147